Two remarkable contributions of Aristotle to the intellectual legacy of the West

D F M Strauss
Northwest-University, School for Philosophy, Potchefstroom Campus
dfms@cknet.co.za

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ABSTRACT

Although it does not need any justification to explore the rich legacy of Greek philosophy, an account of its lasting insights does need lifting out the significant elements of continuity, while disregarding those stances that reflect elements of discontinuity. With a sense of critical solidarity this investigation mainly focuses on two brilliant insights present in the philosophical works of Aristotle.

The first one concerns the legal-ethical principle of equity. Since the universal scope of a law cannot possibly foresee all the unique circumstances that may occur in the future, it does happen that in a particular instance applying the law will lead to an injustice, in which case the applicable law has to be set aside on behalf of equity. However, the law itself cannot be eliminated since it is a condition for a stable legal order.

The second one focuses on the remarkable fact that modern set theory still conforms to the two criteria which Aristotle stipulated for continuity. Aristotle held the view that something continuous must be infinitely divisible and that each point of division must be taken twice, namely as starting-point and as end-point. What appears to be perplexing is that whereas Aristotle rejects the actual infinite, modern set theory accepts it! This apparent contradiction becomes understandable (and is eliminated) once it is realised that the two alternative approaches merely represent different aspectual perspectives: when these two conditions are observed from the angle of approach of the aspect of space, only the successive infinite is needed (surfacing in the infinite divisibility of what is continuous – Aristotle's approach), and when they are accounted for from the perspective of the numerical aspect, the actual infinite (the idea of an infinite totality*) is required (Cantor-Dedekind).*

1. ORIENTATION

One of the distinctive features of philosophy, practically found throughout its history, is that most prominent philosophers have been exceptionally well informed about the state of intellectual endeavours for the time in which they live, in most cases overseeing practically the entire available scholarship of their day. Among them we meet giants, such as Plato, Aristotle, Augustine, Thomas Aquinas, René Descartes, Gottfried Wilhelm von Leibniz, Immanuel Kant, Georg Wilhelm Friedrich Hegel, Arthur Schopenhauer, Wilhelm Windelband, Max Weber, Ernst Cassirer, Ludwig Wittgenstein, Herman Dooyeweerd, Martin Heidegger, Jürgen Habermas, Jacques Derrida and Hans-Georg Gadamer. Intuitively one feels that it is still worthwhile to engage in a serious analysis and interpretation of the ideas advanced by thinkers like these. Although much of their work is *dated* it appears as if their philosophies are never really *outdated*. Perhaps this may be explained by following Van Riessen who characterised philosophy as the science dedicated to *boundary questions* (Van Riessen 1970: 11, 19, 20).

2. THE IMPASSE OF HISTORICISM IN APPRECIATING GREEK PHILOSOPHY

Another reason may be found in the fact that the above-mentioned thinkers, each one in his own distinct way, discovered traits of the world which we still have to account for. Since such factual states of affairs still hold for what we can experience today, it is impossible to bypass them by claiming that they are "outdated." Particularly *historicism* may uphold the thought that older works are "outdated." Only contemporary literature is supposed to be "up-to-date"! However, such a historicist stance contains two obvious pitfalls:

(i) Almost all the names of prominent philosophers mentioned in the previous paragraph will then be *outdated* as well as the historically significant turning points in the development of the various academic disciplines (natural sciences and humanities); and

(ii) Whatever position is assumed in the *present* will just after a few years share the same fate and be considered *outdated* as well. The inherent *relativism* in such a historicist orientation not only uproots the significance of scholarly work, but it also terminates in *nihilism*.

Yet it is striking to note that in many respects scholarship during the last hundred to two hundred years constantly has remained attached to systematic issues and distinctions which emerged in Greek philosophy.

3. THE SIGNIFICANCE OF GREEK PHILOSOPHY FOR MODERN MATHEMATICS AND PHYSICS

3.1 The foundations of mathematics dependent on the Greeks

By the end of the 19th century mathematicians believed to have "conquered" continuity fully in set theoretic terms – in the line of Bolzano, Weierstrass, Dedekind and Cantor. But intuitionistic mathematics opposed these claims while holding on merely to the potential infinite, which is according to them exemplified in the *infinite divisibility* of a continuum. Already in 1921 Weyl declared that having parts is a basic property of the continuum: "and so Brouwer's theory posits this relationship (in accordance with intuition, against which 'atomism' so seriously errs) for the mathematical treatment of the continuum" (Weyl, 1921: 77). A decade later Weyl underscores the importance of Greek thought for an understanding of the foundations of mathematics: "Yes, especially now are we invited everywhere in the foundations of mathematics to go back immediately to the Greeks" (Weyl, 1931: 1)[1].

3.2 Did modern physics exceed Greek thinking on what is matter?

During the fifties of the previous century three books appeared in the series: "Rowohlts deutsche Enzyklopädie," written by Arthur March (1957), Erwin Schrödinger (1956) and Werner Heisenberg (1956). All three discuss the new physics of the 20th century, but it is done while explicitly acknowledging the intimate connection with and importance of Greek thought. Schrödinger refers to Greek thinking in connection with the problem of the continuum as well as regarding the discrete nature of their "atoms." Stegmüller (1987) even states that in spite of our extensive experiential knowledge we are, after 2000 years, not closer to an answer to the question what matter is – keeping in mind that the early Greek

[1] "Ja gerade heute sehen wir uns genötigt, Überall in den Grundlagen der Mathematik wieder unmittelbar auf den Griechen zürückzugehen."

thinkers pursued a "purely speculative foundation." Amidst and in spite of being hidden behind mountains of mathematical formulas, the underlying problem constantly surfacing in the history of our reflections on what matter is, is according to Stegmüller found in two basic conceptions, namely what he calls the *atomistic conception* and the *continuity conception*. These two theoretical approaches attempt to come to terms with two equally lasting classical problems, namely the *immutability* of matter and the apparent unlimited *changefulness* of matter (Stegmüller, 1987: 91)[2].

From a systematic point of view it is clear that the two conceptions (*atomistic* and *continuity*) and the two problems (*persistence* and *change*) actually reveal the co-conditioning role of the four most basic aspects of our experience of the universe, namely the aspects of number, space, movement and the physical. Greek philosophy already explored the options provided by these aspects to approach reality in a theoretical way and to explain whatever there is. In the development of the thought of Plato and Aristotle this growing legacy surely played an important role, even though we have to keep in mind that both contributed in innovative ways to this legacy.

4. WESTERN PHILOSOPHY AS FOOTNOTES TO PLATO

At this point we are reminded of the well-known and frequently quoted remark by Whitehead who declared that the European philosophical tradition could best be characterised as a series of footnotes to Plato (Whitehead 1929: 65).

4.1 From number and space to the meaning of law

Schrödinger explores a significant feature of the Pythagorean theory of numbers and harmony. The two extreme interpretations of the basic thesis of the Pythagoreans are found in the opposing statements: *things are numbers* and *things are like numbers*. Aristotle interprets these "things" in the first place as sensory, material objects and attributed a thing character even to "things" such as the *soul* and *justice* (Schrödinger 1956: 49). Schrödinger also points at the role of square numbers – (1), 4, 9, 16, 25 …. For the Greeks the number

2 The problem of the "*Unvergänglichkeit der Materie* und das der – scheinbaren oder wirklichen? – unbegrenzten Verwandlungsfähigkeit der Materie." Schrödinger holds that applying our idea of the continuum to *energy* is not appropriate (Schrödinger 1956: 83).

1 was not a number at all, which turned the number 4 into the first *square number*. This number (4) is divisible into two *equal* parts ("*gleiche* Faktoren"). Schrödinger immediately relates this feature to our experience of jural relationships, captured in words such as "jural equality" (*Rechtsgleichheit*) and "equal justice" (*gleichberechtigt*) (Schrödinger 1956: 49).

This suddenly opens up a new philosophical issue, the relation between our experience of *legal relationships* and *numerical relationships*. We shall first investigate the Aristotelian view of equity (*epikeina*; *equitas*) and then return to the exceptional insight found in the thought of Aristotle regarding the nature of continuity.

4.2 The Platonic background

With Protagoras (the founder of the school of Sophism), a new conception emerged; one in which the de-divinised fluid nature (*phusis*) was opposed to the law (*nomos*) of the city-state – the latter was supposed to supply human nature with a *human form*. In line with the archaic conceptions of law (*dike*) Plato continued the idea that there is an encompassing cosmic law order. It includes his understanding of the three parts of the human soul in *Politeia* which served for him as the foundation of his view of the state. Having divided the soul into a rational part (the *logistikon*), a spirited part (the *thumoeides*), and an appetitive part (*epithumétikon*), he continued to apply this division to his ideal state, where *wisdom* (*sophia*) as the virtue of the rational part of the soul guides the estate of ruling philosophers, *courage* (*andreia*) is the virtue that directs the guardians, whereas the virtue or *temperance* serves the lowest estate of farmers and labourers. The general virtue of justice, which embraces all the others and maintains them within their confines, thus impacts on the ideal state as a whole (Cornford, 1966, *Politeia*, 443 ff.).

It must be clear that this view of Plato does not acknowledge the basic and distinct legal spheres found in present-day differentiated societies. The two highest political estates partake in communal ownership at the expense of private marital and family life. Likewise, the lowest estate is stripped of all rights, while children are subjected to an educational programme regulating their lives in detail – all in all a totalitarian structure. Later on Plato developed his idea of a state ruled by laws, but just like Aristotle he continued to be oriented to the all-encompassing legal power of the city-state (the *polis*). Ultimately Plato therefore advanced a totalitarian view of the Greek city-state (*polis*), an outcome directed by the form motive of Greek culture.

5. MATTER AND FORM AS OPPOSING PRINCIPLES OF ORIGIN: THEIR IMPACT UPON ARISTOTLE'S VIEW OF THE *POLIS*

Aristotle struggled with the dualism entailed in the two principles of origin accepted by him, namely the *matter* principle of potentiality and the *form* principle of actuality[3]. As a "political animal" (*zoon politikon*) the human being inherently disposes over a rational-ethical *essential form* and this essential form can only come to completion (fulfilment) within the *polis* (the city-state).

Within an organicistic thought-scheme Aristotle viewed marriage, the nuclear family and the extended family merely as a means to the goal of forming *good citizens* of the state. The family-community is identified with an economic social entity which not only embraces the relationship between husband and wife, but also that between parents and children and in addition it also includes the relation between master and slave (*Politics*, Book I, Chapter III, Aristotle 2001: 1130-1131).

6. JUSTICE AND EQUITY

In his earlier works Aristotle did not restrict *justice* to human societal relationships[4]. Both he and Plato gave expression to the primacy of the form motive in their thought by employing the concept *taxis* (order), which, for Aristotle, in his *Politica*, is seen as the ontic form (*eidos*) of the state. In Book III of the *Politica* (Chapter 15) Aristotle contemplates the question whether it is better "to be ruled by the best man or by the best laws" (*Politica* 1286a8-9; Aristotle 2001: 1199). Since law only speaks in universal ("general") terms, it "cannot provide for circumstances" (*Politica* 1286a10-11; Aristotle 2001: 1199), from which it follows that a "government acting according to written laws is plainly not the best" (*Politica* 1286a15-16; Aristotle 2001: 1199).

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3 Happ points out that primary matter (*prima materia* – the highest matter principle) is the counter pole of the highest form (Happ 1971: 562, 696-697).

4 Whether or not the *Magna Moralia* (*Great Ethics*) should be attributed to Aristotle is controversial (though already Werner Jaeger defended the view that it contains extracts from the *Eudemian Ethics* and *Nicomachean Ethics* by someone else). In the last part of *Magna Moralia* the human soul is divided into a rational and a non-rational part. Justice now assumes the meaning of obeying the law (*nomos*) as complete virtue without being restricted to fellow humans.

In his *Nicomachean Ethics* Aristotle further explores this perspective advanced in the *Politica* by distinguishing between general justice and particular justice and by applying the relation to the other also to general justice. This is distinguished from particular justice with its concern for the well-known yardstick of *distributive* (*geometrical*) equality and *commutative* (*arithmetical*) equality.

6.1 Is equity the justice of law?

The universality entailed in written laws is questioned because the law-giver cannot (in advance) oversee all the unique circumstances that may occur. Aristotle clearly realised that this shortcoming requires a flexibility exceeding the strict application of laws. This flexibility cannot be applied *in general* because then the ordering function of law will be eliminated and chaos will prevail. It must always be directed merely to a unique (unforeseeable) situation: "Yet surely the ruler cannot dispense with the general principle which exists in law" (*Politica* 1286a16-17; Aristotle 2001: 1199-1200).

How does Aristotle elucidate his view of equity (*epikeia*) in Book V, Chapter 10 of his *Nicomachean Ethics* (Benns 2009)? First of all, when the state makes a law, the intent is general – it is supposed to be applicable to all possible occurrences regulated by it. Yet, the universality of the law always has to "meet" what is unique and particular in each individual case – and it may happen that applying the law in a particular case causes an injustice. Del Vecchio points out that in every individual legal case the jurist has to lift out those essential elements which are significant in a jural sense by disregarding the non-essential features (Del Vecchio 1951: 388-389).

Aristotle remarks that "justice and equity are not absolutely identical, yet cannot be classified as different." Although he holds that equity is "a higher thing than one form of justice" it "is itself just and is not generically different from justice." Nonetheless the puzzling feature of equity is that although it is just, it is not the *justice of the law*.

6.2 Setting aside the (universal) law solely in a particular (individual) instance

What Aristotle here has in mind is that applying a law to unique circumstances may lead to an injustice. Rectifying this shortcoming of a given law highlights the difference between what is universal and what is individual. Aristotle views this rectification as a "method of restoring the balance of justice when it has been tilted by the law." Setting an applicable law aside in such a rectifying way is therefore still *just* – but obviously it does not comply with what is stipulated in the law. Aristotle argues that the "need for such a rectification arises from the circumstance that law can do no more than generalize" and according to him "there are cases which cannot be settled by a general statement." A general statement "cannot exclude the possibility of error" and for this reason law cannot be *just* in all instances.

Rectifying a shortcoming in a law *on behalf of equity* (*ex equitate*) does not intend to *violate* the law. Its intention is to set the law aside – yet not in general but only in one or another unique situation[5]. Del Vecchio affirms this when he explains that equity in no way intends to break the law since its sole intention is to contribute to the realisation of what the laws aims at, albeit in a *more perfect way* (Del Vecchio 1951: 389). Aristotle is therefore justified in his explanation that setting a law aside on behalf of equity does not mean that such a law is *bad*: "Yet that does not make it a bad law, the error lying not in the law or the lawgiver but in the nature of the case; the data of human behaviour simply will not be reduced to uniformity" (*Nicomachean Ethics*, Book V, Chapter 10; Aristotle 2001: 1019-1020).

Although both Plato and Aristotle adhere to a totalitarian view of the *polis* which in fact does not delimit the competence of the city-state, his understanding of equity provided Western thought with an insight of lasting value, because equity is still honoured in many legal systems as a legal-ethical principle of justice, alongside other principles of juridical morality (such as *bona fides*, and *fault*). Berman refers to the Graeco-Roman name *aequitas* in connection with "canonical equity" which is meant to mitigate the strict law in exceptional cases where "good faith, honesty, conscience, or mercy so required" (Berman 1983: 196).

5 Cotterrell quotes Proud who points out that the distinction between law and discretion (equity) pretty much runs through legal history: "almost all of the problems of jurisprudence come down to a fundamental one of rule or discretion … both are necessary elements in the administration of justice … there has been a continual movement in legal history back and forth between wide discretion and strict detailed rule, between justice without law, as it were, and justice according to" (Cotterrell 1984: 172-173).

6.3 The ultimate motivation and the cultural setting of Aristotle's conceptions

Of course appreciating the lasting value of equity in the thought of Aristotle does not entail that his view took shape divorced from the ultimate underlying basic communal inspiration of Greek culture which is given in the matter motive of the formless stream of life and the cultural motive of form, measure and harmony. The late-Aristotelian conception of equity, found in the *Nicomachean Ehtics*, is based on the assumption that human measures of justice and law are necessarily imperfect. Although the form motive motivates the idea of the rational-ethical ontic form of human nature, the realisation of ethical norms participates in the principle of eternal flow which causes Aristotle to hold that within the practical domain of ethics there is not anything firm and immovable. This includes the variability of human conceptions of justice. Aristotle points out that the term *ethike* contains a slight variation from the word *ethos* which means *habit*. "From this it is also plain that none of the moral virtues arises in us by nature; for nothing that exists by nature can form a habit contrary to its nature" – and ethical standards also merely rest on statutory law and custom since they have no foundation in nature (*Nicomachean Ehtics*, Book II, Ch.1, 1103a18-20; Aristotle 2001: 952). This position explains why Aristotle holds that ethical norms do not have a validity without exceptions, although he does not defend a *relativistic* position.

Dooyeweerd explains that for Aristotle equity must be viewed as a form of *particular justice* serving the improvement of positive statutory law in those particular instances where it would cause unjust consequences. Since equity for Aristotle is a virtue directed at natural true law, it does not stand in opposition to law as such – it is only opposed to what is imperfect in positive statutory law. He continues by pointing out that already in the *Magna Moralia* equity was related to "well-meant insight" (*eugnomosunè*) as a special *cognitive faculty*. However, an element of criticism is here in place, because Aristotle relates his equity principle to natural law and restricts the equity principle to the individual case (in casu) and therefore does not allow it to function as a principle of order, such as "pacta legitima sunt servanda" (lawful agreements ought to be kept). In addition it should be noted that only when the jural and moral aspects of our experiential world are appreciated in their uniqueness and mutual coherence will it be possible to bring the long-standing dispute regarding the function of equity in law to a satisfactory solution (Dooyeweerd 1958: 48-50).

The comprehensiveness of the legacy of Aristotle cannot be better illustrated than by moving to a totally different domain, namely the mathematical criteria for continuity. The remarkable fact is that modern and contemporary set theoretical approaches still employ the criteria stipulated by Aristotle!

7. ARISTOTLE ON CONTINUITY

We can now return to Schrödinger's earlier remark about the connection between "jural equality" (*Rechtsgleichheit*) and "equal justice" (*gleichberechtigt*). The notion of *equality* serves as a link between our awareness of *space* and the meaning of the *jural*. Although our experience of space almost always generated the idea of *continuous extension*, it turned out that it seems pretty difficult to explain the meaning of continuous extension. Dantzig realised this quite clearly:

> *From time immemorial the term continuous has been applied to space, ..., something that is of the same nature in its smallest parts as it is in its entirety, something singly connected, in short something continuous! …don't you know .... any attempt to formulate it in a precise definition invariably ends in an impatient: 'Well, you know what I mean!'* (Dantzig 1954: 167).

7.1 A discrete quantity and a continuous quantity

One may see Aristotle, in spite of some brief earlier traces found in the thought of Anaxagoras – see the latter's B. Fragments 3, 6, and 8 (Diels & Kranz 1959: 231-240) – as one of the first authors who rejected the idea of a line as a continuum of points (Becker 1964: 7, 70). Aristotle explored this view further where he distinguishes between a *discrete quantity* and a *continuous quantity* (Aristotle 2001: 14) and where he stipulates that in the case of number, as a discrete quantity, no "common boundary" is present among the parts.

In his *Physica* Aristotle provides a broader explanation of discreteness. Things in succession entail that "there is nothing of their own kind intermediate between them." The order of a line is therefore given in point-line-point-line-point-line. In 1933 Ludwig Fischer still argued that between two non-coinciding points we always have "continuum," owing to the fundamental law: "continuum-point-continuum" (Fischer 1933: 86-87). Aristotle infers from this position that a continuous whole is *infinitely divisible*, for according to him it is impossible to explain the continuity of a straight line in terms of its points. His objection states: “Moreover, it is plain that everything continuous is divisible into divisibles that are infinitely divisible: for if it were divisible into indivisibles, we should have an indivisible in contact with an indivisible, since the extremities of things that are continuous with one another are one and are in contact” (*Physica*, 231a29-31; *Physica*, 231b15ff., Aristotle 2001: 316). That “which is intermediate between points is always a line” (Aristotle 2001: 316).

7.2 Aristotle introducing the potential infinite

In order to overcome the problems posed by Zeno's paradoxes, Aristotle introduced his notion of the potential infinite (also *De gen. et corr*. 316a14 ff.). The infinite divisibility of a line is granted by him, with the qualification that this divisibility is only potential and can therefore never be carried through actually. Furthermore, Aristotle upholds that no moving object is “counting” while it moves, since then the objection of Zeno would have been valid: in order to traverse a finite spatial distance it is required to actually count an infinite sequence of numbers.

Aristotle writes: “In the act of dividing the continuous distance into two halves one point is treated as two, since we make it a starting-point and a finishing point.” To this he adds that the infinite number of halves thus obtained is never actual but only potential halves (*Physica* 263a22-25; Aristotle 2001: 383).

7.3 Aristotle’s rejection of the actual infinite

Aristotle firmly rejects the actual infinite, mainly in terms of the following objections (*Phys.* 204a20 ff., *Metaf*. 1066 b 11 ff., and *Metaf*. 1084a1 ff.):

(i) if the actual infinite is composed out of parts, every one of these parts would be , taken by itself, actually infinite, implying the absurdity that the whole is no longer prior to (or greater than) a part (*Pol*. 1253a19-20);

(ii) if the actual infinite has finite parts we would be able to perform the impossible by counting the infinite, or else there must exist transfinite numbers being neither even or odd[6].

7.4 The two criteria stipulated by Aristotle for continuity

Aristotle thus postulated two criteria for continuity: (i) it must be infinitely divisible and (ii) every point of division should be taken twice (as end-point and as starting-point). The remarkable fact is now that the modern set theoretical approach (Cantor-Dedekind) to continuity stipulates the *same* two criteria. Cantor explains that his view of a *perfect set* is equivalent to Dedekind's cut theorem (Cantor 1962: 194). Böhme demonstrates that the two stipulations contained in Cantor’s definition of the continuum meet the two above-mentioned conditions posited by Aristotle, namely *coherence* and a characteristic which ensures the existence of dividing points for infinite division – where such points of division are taken twice (Böhme 1966: 309). By allowing only a Dedekind-cut at divisions, Böhme explains:

> *… when a Cantorian continuum as such is divided into two by means of the indication of a point such that the one set contains those points which are in numerical value greater than or equal to the indicated point, while the other set contains those points of which the numerical values are smaller than or equal to the numerical value of the indicated point, both parts are again continuous. Such divisions are possible into infinity (due to the perfection of the continuum), and the parts are still coherent in the Aristotelian sense (i.e. their limit-points are the same)* (Böhme 1966: 309)[7].

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6 It turned out that both objections actually highlighted two essential features of the actual infinite (Cantor 1962: 182-183; Dedekind 1887: 13-14)! That these two objections are in fact depicting characteristic features of the so-called actual infinite would not be further discussed in this context.

7 “Teilt man ein Cantorsches Kontinuum durch Auszeichnung eines Punktes so in zwei Teilmengen, dass zu der einen Menge die Punkte gehoren, deren Zahlenwert grosser oder gleich dem Zahlenwert des ausgezeichneten Punktes ist, und zur anderen die Punkte, deren Zahlenwert kleiner oder gleich dem Zahlenwert des ausgezeichneten Punktes ist, so sind die Teile wiederum Kontinua. Solche Teilungen sind in infinitum moglich wegen der Perfektheit des Kontinuums, und die Teile sind stets im Sinne der 1. Definition des Aristoteles zusammenhangend, d.h. ihr Äußerstes, nämlich der Grenzpunkt, ist eines.” These issues recur in the recent work of JL Bell (2008: 5 ff.).

This is an exceptional outcome: the continuum as defined by Cantor-Dedekind presupposes the employment of the actual infinite (the set of real numbers) but nonetheless conforms to the two requirements for a continuum stipulated by Aristotle, which reject the actual infinite. How can we explain this perplexing situation – were Cantor and Dedekind employing and rejecting the actual infinite at the same time or did Aristotle actually *use* the idea of the actual infinite *implicitly*? Alternatively, is the Cantor-Dedekind definition in the last instance not *purely arithmetical* in nature? In answering this question our specific appreciation of the lasting effect of Aristotle's approach to continuity will be assessed.

The crucial question is: how is it possible that Cantor and Dedekind employ the same two criteria as Aristotle (infinite divisibility and taking any point of division twice [as a starting-point and as an end-point]), but still oppose each other regarding the nature of infinity?

This almost perplexing situation is immediately clarified through a brief analysis of the meaning (uniqueness and coherence) of the aspects of number and space.

An endless succession of numbers, such as 1, 2, 3, …, belongs to the primitive meaning of number and constitutes, as noted above, what Aristotle designates as the *potential infinite*. It represents the literal quantitative meaning of the infinite, a succession without an end. However far any succession is extended it can still be continued without an end. It is only when we choose the spatial aspect as our point of orientation that the meaning of succession is "turned inwards," such as when the *infinite divisibility* of a continuum is considered, as we pointed out earlier. The intuitionist mathematicians, Brouwer and Weyl, both defend the view that the whole-parts relationship is characteristic of the continuum. The fact that the idea of wholeness derives from the Greek word *holon* and the idea of a totality from Latin (*totum*), shows that the terms *totality* and *whole* are synonymous.

Wieland points out that this definition encompasses two related but distinct features of continuity in the thought of Aristotle, namely a *property* and the *expression of a relationship* (Wieland 1962: 284-285) while Von Weizsäcker underscores the mere *potentiality* of on-going counting, further *dividing* and *extending* magnitudes (infinity is only potential and not actual) (Von Weizsäcker 2002: 75). Let us consider in some more detail the nature of continuity, parts, and dividing as well as Aristotle's conviction that nothing continuous is divisible into things without parts (*Physica* 232 a 23, *Physica* 231 b 10-12; Aristotle 2001: 316-318).

8. INFINITE DIVISIBILITY AND AN INFINITE TOTALITY: UNDERSTANDING THE AGREEMENT AND DIFFERENCE BETWEEN ARISTOTLE AND CANTOR-DEDEKIND

Interestingly a non-intuitionist mathematician, such as Paul Bernays, who was the co-worker of the well-known German mathematician David Hilbert, underscores the fact that the property of being a totality is characteristic of the continuum and that it obstructs every attempt to arithmetise it. He writes: “The idea of the continuum is a geometrical idea which analysis expresses in terms of arithmetic” and on the same page adds the remark: “And it is this feature [the property of a totality] which opposes a complete arithmetization of the continuum” (Bernays 1976: 74).

It should be noted that in the case of something continuously extended the focus of Aristotle is not on parts per se, but on *dividing*, combined with the equally important emphasis on the fact that dividing what is continuous constantly generates parts which are once again divisible. The parts of a continuum are therefore once again continua which mutually cohere in a continuous way. Although this remark may sound circular, Wieland explains that a certain kind of circularity is always encountered in respect of irreducible basic concepts (Wieland 1962: 288). On the preceding page Wieland also speaks of the “irreducibility of the continuum”. The reason why a line cannot be seen as a continuous coherence of points, according to Aristotle, is not found in the nature of a line, but rather because it contradicts the *nature of points*. Since points are not extended they do not have extremities, which means that instead of cohering continuously they will collapse into one point (without any extension). Moreover, if a continuum consisted out of indivisible points, then it must be possible to divide a line into points, which would mean that continuity is no longer divided in parts that are *continua* in their own right, allowing for a continuing process of division. Aristotle writes:

> *Again, if length and time could thus be composed of indivisibles, they could be divided into indivisibles, since each is divisible into the parts of which it is composed. But, as we saw, no continuous thing is divisible into things without parts.* (Aristotle 2001: 316-317).

Given his informed understanding of the thought of Aristotle it is rather surprising that Wieland would remark that the Aristotelian concept of the continuum has little in common with modern mathematics (Wieland 1962: 287).

What Aristotle actually highlighted is that one can stipulate, from the perspective of the spatial aspect, two criteria for continuity that are still valid within the context of a set theoretical analysis. The sole difference is that since the set theoretical approach to continuity proceeds from the perspective of the numerical aspect, it explored the idea of an *infinite totality* (since Aristotle known as the actual infinite).

Hilbert (1925) provides the following explanation of the difference between the potential and actual infinite:

> *If one briefly wants to explain the new Cantorean understanding of infinity it could be said: in analysis we only have the infinitely small and the infinitely large available, as limit concept, as something becoming, originating, brought forth, that is, as it is said, that is potential infinity. But this is not infinity in its genuine sense. The latter is given when we observe, for example, the totality of the numbers 1, 2, 3, 4, … itself as a completed unity or when we see the points of a line as a totality of entities which are at hand as something completed*[8]*.*

The idea of an infinite totality clearly enriches the meaning of number by “borrowing” from space the feature of viewing a succession of numbers as a completed totality. In the configuration of an infinite totality we therefore meet, within the arithmetical aspect, an “imitation” of the original (and irreducible) meaning of space.

8 “Will man in Kürze die neue Auffassung des Unendlichen, der Cantor Eingang verschafft hat, charakterisieren, so könnte man wohl sagen: in der Analysis haben wir es nur mit dem Unendlichkleinen und dem Unendlichengroßen als Limesbegriff, als etwas Werdendem, Entstehendem, Erzeugtem, d.h., wie man sagt, mit dem potentiellen Unendlichen zu tun. Aber das eigentlich Unendliche selbst ist dies nicht. Dieses haben wir z. B., wenn wir die Gesamtheit der Zahlen 1, 2, 3, 4, ... selbst als eine fertige Einheit betrachten oder die Punkte einer Strecke als eine Gesamtheit von Dingen ansehen, die fertig vorliegt. Diese Art des Unendlichen wird als aktual unendlich bezeichnet” (Hilbert 1925: 167).

9. SUMMARY

Given the historical distance between Greek philosophy and the contemporary scene within the field of philosophy it stands to reason that multiple elements of Greek philosophy might be regarded as outdated. However, with a sense of critical solidarity the present article investigated two important insights which surfaced in the thought of Aristotle – in order to show that notwithstanding the cultural setting of Greek philosophy there are still significant distinctions and insights which we can appreciate today.

Current legal practices in the West still apply the legal-ethical principle of equity while standard set theoretical views still conform to the two criteria stipulated by Aristotle for continuity (namely infinite divisibility and taking every point of division twice, as starting-point and as end-point). The apparent perplexing situation, where Aristotle rejects actual infinity and modern set theory accepts it, is understandable as soon as it is realised that the two mentioned criteria are merely observed from *different aspectual perspectives*: when these conditions are observed from the aspect of space the successive infinite, surfacing in the infinite divisibility of what is continuous, is only needed (Aristotle's approach), and when they are accounted for from the perspective of the numerical aspect the actual infinite is required (where the idea of an infinite totality represents a numerical imitation of the totality character of spatial continuity – Cantor-Dedekind).

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