Primitive terms and the limits of conceptual understanding[1]

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Ignoring primitive terms leads to an infinite regress. The alternative is to account for an intuitive understanding (immediate insight) into the meaning of such terms. The current investigation proceeds on the basis of an idea of the structure of the various modes of being (modi/modalities) within which concrete (natural and social) entities function. Examples of primtve terms are given from disciplines such as mathematics, physics and logic and they are related to the general idea of a modal aspect. It is argued that primitive terms are not isolated but reveal their meaning only through their interconnections with other primitive terms that are embedded in other modal aspects. However, although primitive terms are found within the various aspects, the meaning of an aspect only comes to expression through its coherence with other aspects, evinced in modal analogies that are qualified by the core meaning of an aspect. There appears to be two options, either reduce what is irreducible or merely provide (partial) synonymous terms for given primitives. The former happens when other unique (primitive) terms are used to define a specific one and the latter when the attempted 'definitions' revert to terms with which the original terms could be meaningfully replaced. It is been pointed out that the coherence between primitives invites every academic discipline to account for the meaning attached to the analogies of primitive terms it is employing, without exploring this additional theme any further.

Introductory remarks

This article intends to approach the problem of primitive terms from a systematic perspective. It aims at discerning important features of such terms and to relate them to a more comprehensive understanding of the importance and inevitability of primitive terms within scholarly universes of discourse, in particular related to the additional perspective of the way in which they mutually cohere. The existence of unique primitive terms calls for an account of their mutual coherence. However, we shall argue that it is incorrect merely to contemplate the idea of the *coherence of primitives* (as suggested by Kurt Gödel), because the problem of primitive terms should be embedded in a general theory of functional aspects or modal aspects. The expression 'modal aspect' is used in the sense of *modes of being* that relate to the *how* of our experience and not to its concrete *what*. The idea of an 'aspect' explores a metaphorical understanding of spatial terms, such as *facets* (from French), *sides* (from many-sidedness – like a diamond) or *nuancefulness* and the term 'modal' is derived from the Latin root 'modus', which indicates a *way* (*mode*) of being or of functioning, such as what is captured in the Latin expressions 'modus vivendi' or 'modus quo' (a way of acting/functioning or a way of being). A chair, for example, functions within various aspects: within the numerical aspect it functions as *one* piece of furniture, within the spatial aspect it functions as an object with a particular size and shape, within the kinematic aspect of motion it

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functions as a moving object,[2] within the physical aspect as an entity with a particular mass and the required physical strength to hold normal people sitting on it, and so on.

Anticipating the conclusion of this investigation it may therefore be stated in advance that the examples that will be discussed and the quotations that will be given are mentioned solely in service of understanding the issue of uniqueness and coherence within the context of a systematic perspective on the nature of *modal aspects*. The latter problem could also be designated as that of the 'coherence of irreducible aspects.' The method used is not synchronic but diachronic. For example, when it is noted what Aristotle, Hegel, Marx or Janich said, the focus will be on an element of their thinking that is significant for a current systematic understanding of the problem of primitive terms. How these thinkers, at different stages of their development, might have altered their views or how someone else later on criticised their position will therefore be left aside.

The aim of this article is therefore to establish what it is that such thinkers have discovered at that specific point in time – a state of affairs that we still have to recognise today even though we may not want to account for it in the same way. What is employed is a diachronic method that uses examples for a systematic purpose. To phrase it slightly differently: when a scholar holds a view that contains an element that directly relates to an understanding of the uniqueness of primitives and their mutual coherence, the other elements as well as possible changes later on found in the intellectual development of that thinker is no longer relevant for this systematic diachronic aim. When Aristotle says that an infinite regress – which is logically unsound – will follow when it is attempted to define everything, one is epistemically justified solely to lift this element out. For our systematic aim stating in addition what one finds unacceptable in Aristotle's philosophy is redundant. Likewise, if no point of connection of the idea of modal aspects (or rationality) advanced in this article is found in the thought of an analytical philosopher such as Kripke one may turn to the thought of the continental analytical philosopher, Wolfgang Stegmüller, where such points are found.

When one-sided points of view are quoted the goal is not to criticise the oversimplified perspective presented by them, but to find out what they discovered that we still have to account for. The arithmeticism of the Pythagoreans did see key features of number still requiring a systematic account from us today, even though we no longer have to reduce mathematics (and reality) to fractions.

In order to ensure that the examples and quotations that are highlighted below are not seen as unrelated, it is necessary briefly to explain in advance how they relate to the general idea of modal aspects, which is the underlying conjecture of this investigation.

Throughout its history philosophy has wrestled with the distinction between entities and properties, sometimes designated as substances and functions (see Cassirer 1910). Greek–Medieval philosophy by and large gave prominence to the substance concept and modern philosophy since the Renaissance assigned primacy to the function concept (the concept of relationships). As explained above, in our everyday experience 'what' questions relate to concrete entities and 'how' questions to the modes of being of reality, to the various aspects.

What is novel in this approach is that reflection on the nature of primitive terms could be embedded in a general theory of functional modes of reality. The conjecture is that the core primitive (indefinable) elements characteristic of the uniqueness of each aspect are embedded within the general modal structure of these aspects.

In support of this general understanding of aspects with their primitive meaning-nuclei, seemingly unrelated examples may be highlighted because they are all related to the structure of a modal aspect. Neither analytical philosophy nor the mainstream continental philosophy (existential philosophy, personalism, phenomenology, developments within the philosophy of science, and postmodernity) developed a theory of modal aspects.

The discussion below will draw upon the ideas of thinkers who discovered elements of the general idea of a modal aspect without developing such a theory themselves. This idea conjectures

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2   Although we may think that the chair is 'at rest,' we have to be reminded of what Einstein wrote to Moritz Schlick (7 June, 1920): 'Rest is a dynamic event in which the velocities are constantly zero, one that for our consideration is, in principle, equivalent to any other event of motion' (Einstein 2006: p. 186).
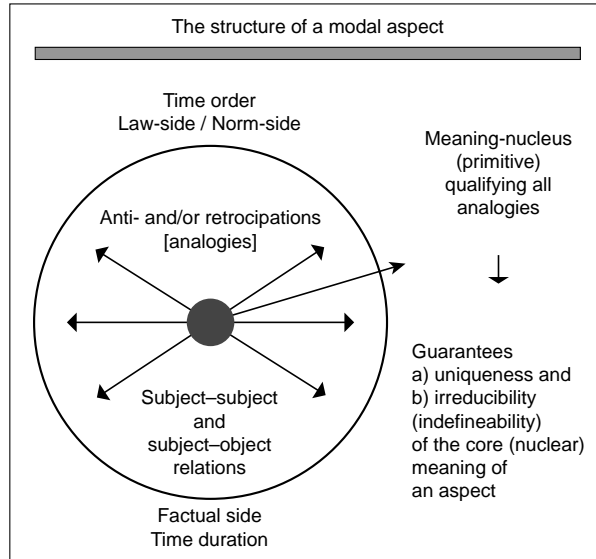
Figure 1.

that every unique modal (functional) aspect has a *meaning-nucleus* that qualifies all *analogical references* to other aspects. At the factual side of these various aspects one also finds subject–subject and subject–object relations[3], which are correlated with the law-side of each aspect (also expressed in the correlation of *time order* and *time duration*). Provisionally, we therefore conjecture that the general structure of a modal aspect embraces the following features.[4]

In the light of the correlation of law and what is factually subjected to it one may also designate an aspect as a *sphere of laws*. Clouser holds that a modal aspect is 'a basic kind of properties and laws' (Clouser 2011: p. 221).

Figure 1 suggests that the uniqueness and irreducibility of an aspect is guaranteed by its primitive core meaning. We proceed by focusing on this primitive meaning followed up by examples of how this primitive meaning relates to other structural features of a modal aspect.

Is the generally accepted view that attempted definitions of what is unique and primitive (indefinable) will end up in an infinite regress sound?

An infinite regress?
Within every philosophical tradition one is confronted with *basic distinctions* and *specific meanings* attached to *particular terms*. In general, the expectation exists that scholars, i.e. scientists in the broad sense of the term, should be able to delimit their intellectual pursuits conceptually. Such a delimitation of a field of investigation is always in need of proper definitions. The problem is that eventually every definition gets stuck in *indefinable terms*. We shall argue that all attempts to move beyond what is truly primitive either terminate in begging the question or run into an infinite regress. A further complication is found in accounting properly for the connections or relationships holding between primitives. The reality of this state of affairs is not restricted to any specific

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3   Within the quantitative aspect there are only subject–subject relations because the relationship between numbers that are added, multiplied, subtracted or divided (expressed in fractions) invariably yields different kinds of numerical subjects, subject to the operations of addition, multiplication, subtraction and division on the law-side of the arithmetical aspect.

4   In the course of our unfolding argument the various structural elements of a modal aspect will be highlighted, except for the distinction of time-order and time duration. One example will suffice. The homogeneity of physical time is absent in living entities because the time phases of birth, growth, maturation, ageing and dying are accelerated – the older a living entity gets, the quicker the process of ageing occurs.

scholarly discipline. It is also not new, because Aristotle already realised that educated thinkers would know 'of what things one should demand demonstration': 'For it is impossible that there should be demonstration of absolutely everything' (there would be an infinite regress, so that there would still be no demonstration) (Aristotle 2001: p. 737, Metaph. 1006a8–10).

Ewing refers to G.E. Moore who believed that *the good* has no definition. 'I think we shall see that some terms must be indefinable if anything is to be defined at all' (Ewing 1962: p. 87). In a different context also Korzybski underscores this fact:

> We thus see that all linguistic schemes, if analysed far enough, would depend on a set of 'undefined terms'. If we enquire about the 'meaning' of a word, we find that it depends on the 'meaning' of other words used in defining it, and that the eventual new relations posited between them ultimately depend on the…meanings of the undefined terms, which, at a given period, cannot be elucidated any further (Korzybski 1948: p. 21).

Recently, Janich argued that from a metascientific perspective one has to start somewhere. 'Therefore it is said that; the demand to define all basic concepts of a theory cannot be accomplished; for every definition requires once more a *definiens*, through which the quest for a definition of all terms of a theory must lead to an infinite regress (Janich 2009: p. 26).

Of course, whatever is accepted as primitive or indefinable still needs to be *designated* by a word or phrase. This assignment of words is not fully arbitrary, because it must call forth some or another kind of an intuitive insight – our concern in the next paragraph.

Intuitive insight?

Yourgrau, for example, explains that Gödel 'insisted that to know the primitive concepts, one must not only understand their relationships to the other primitives but must grasp them on their own, by a kind of "intuition"' (Yourgrau 2005: p. 169).

It should be noted, however, that this statement does not realise that primitive terms are not primitive *concepts*. We need a *word* (a 'primitive term') to *designate* our *intuitive insight*, but the latter exceeds the limits of concepts. Moreover, what is primitive appears to be indefinable, which is equivalent to being *irreducible*.

The just-mentioned remark of Yourgrau in respect of a kind of 'intuition' opens up the way to a reflection on the role of *evidence* as well. The gifted German mathematician, David Hilbert, believed that it will be possible to provide a formal proof for the consistency and completeness of an axiomatic system, but the youthful genius of Gödel, at the age of 25, ruined this dream in 1931. Hilbert died in 1943 and three years later his student Hermann Weyl wrote (with a view to the outcome of Gödel's proof and the inevitability of taking into account what is *evident*), 'It must have been hard on Hilbert, the axiomatist, to acknowledge that the insight of consistency is rather to be attained by intuitive reasoning which is based on evidence and not on axioms' (Weyl 1970: p. 269).

Also, the (continental analytical) philosopher of science, Wolfgang Stegmüller, argues that the basic nature of evidence ought to be accepted as a prerational original decision [*vorrationale Urentscheidung*] that has to be taken in each singular case (Stegmüller 1969: pp. 168–169). This forms the basis for his conclusion that there is no single domain in which a self-guarantee of human thinking exists – one already has to believe in something in order to justify something else (Stegmüller 1969: p. 314).

Therefore, accepting that there are primitive terms avoids an infinite regress and requires an intuitive insight based upon what is evident. We now proceed by reflecting upon [such] unique terms.

Unique terms

The most basic quality that we can discern in respect of anything is its *distinctness*, that it differs from everything else even when something else belongs to the same category or kind of entities. One may also refer to this state of affairs by speaking of a *distinct identity*. Being individually distinct cannot be asserted apart from an awareness of the primitive meaning of number (discrete quantity) and numerical succession. Loux formulates his *Principle of Constituent Identity* therefore in such a way that the *quantitative foundation* in its coherence with *identity* is revealed – compare

the phrase: *numerical identity* (see Loux 2002: p. 113). Something is always *one* amongst *many* others – of the same or a different kind.[5] This gives rise to the question: how do we understand the meaning of the one and the many?

In search of an answer to this question we will focus on the key role played by terms derived from the primitive meaning of the aspects of number and space – *multiplicity*, *succession*, *wholeness* and what is given *at once* (*simultaneously*).

From multiplicity to wholeness

The age-old human response to a given multiplicity is to 'conquer' it by counting *how many* there are. Whoever engages in an act of counting pursues a particular *order of succession*. The numerical order of succession constitutes the most basic assumption of our understanding of number. This order of succession forms the backbone of the principle of (mathematical) induction and it prompted Weyl to state that from an intuitionistic point of view it is 'complete induction' that safeguards mathematics from being an enormous tautology. To this he adds that this principle characterises the assertions of mathematics to be synthetic and not analytic in nature (Weyl 1966: p. 86).

The meaning of the numerical awareness of a discrete multiplicity lies at the basis of the perennial philosophical problem of *unity and diversity*. It prompted Russell to argue that the 'opposition of identity and diversity in a collection constitutes a fundamental problem of Logic – perhaps even the fundamental problem of philosophy' (Russell 1956: p. 346).

The idea regarding the *unity* of a multiplicity eventually resulted in a key element of the modern notion of a *set* as it was developed by Cantor. However, he included a numerical and a spatial element in his circumscription of a set: (1) a *multiplicity* of elements and (2) collecting them into a *whole*. Cantor holds that 'Under the term "set" we understand every collection *M* of determinate and properly distinct objects m of our intuition or our thinking (which are called the "elements" of *M*) into a whole' (this formulation appeared originally in Cantor 1895 – see Cantor 1962: p. 282; translation DFMS).[6]

Recently, the French philosopher Badiou in connection with Cantor's set concept remarks, 'A set, in Cantor's sense of the word has no essence besides that of being a multiplicity; it is without external determination because there is nothing to restrict its apprehension with respect to something else; and it is without internal determination because what it gathers as multiple is indifferent' (Badiou 2005: p. 46).

However, when we compare this remark with Cantor's definition of a set, it is clear that Badiou left out a key element of it, for he highlights only one of two key elements in Cantor's definition. For Cantor a set embraces both the primitive meaning of number (a multiplicity of elements) and the notion of wholeness (totality/*Ganzheit*).

Badiou does not account for the latter part of Cantor's definition. The crucial question is why Cantor had to introduce this second key element in his definition, namely the idea of *wholeness* or a *totality*? Is it perhaps because the whole-parts relation is original and irreducible?

This may be concluded from the views of Cantor, Russell, Gödel and Bernays, to mention three prominent mathematicians and a famous logician. Cantor does it in his just-mentioned emphasis on *Ganzheit* while Russell, Gödel and Bernays recognised key structural features of space.

Russell holds that 'The relation of whole and part is, it would seem, an indefinable and ultimate relation' (Russell 1956: p. 138).

Bernays claims 'that the idea of the continuum originally is a geometrical idea, expressed by analysis in an arithmetical language. In addition he believes that the *totality character* of continuity stands in the way of every attempt to arithmetize the continuum fully' (Bernays 1976: pp. 74, 187–190).

Gödel defends a related view in his understanding of sets.

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5 A full account of this state of affairs will take us beyond the confines of this article to the distinction between concep-
   tional knowledge and concept-transcending knowledge.

6 'Unter einer "Menge" verstehen wir jede Zusammenfassung *M* von bestimmten wohlunterschieden Objekten unserer
   Anschauung oder unseres Denkens (welche die "Elemente" von *M* genannt werden) zu einem Ganzen.'

The operation 'set of x's' (where the variable 'x' ranges over some given kind of objects) cannot be defined satisfactorily (at least not in the present state of knowledge), but can only be paraphrased by other expressions involving again the concept of set, such as: 'multitude of x's', 'combination of any number of x's', 'part of the totality of x's', where a 'multitude' ('combination', 'part') is conceived as something that exists in itself, no matter whether we can define it in a finite number of words (so that random sets are not excluded). (Gödel 1964: p. 262)

Gödel also once said: 'that sets are "quasi spatial" – to which Wang added the remark: "I am not sure whether he would say the same thing of numbers"' (Wang 1988: p. 202).

The general perspective is that the whole-parts relation is basically spatial in nature because the original meaning of the term 'whole' (synonymous with a *totality*) has its original seat within the spatial aspect. Cantor's set theory is therefore a *spatially disclosed* arithmetical theory. While intuitionistic mathematics rejects the idea of an *infinite totality*, Cantorean and axiomatic set theory explicitly employs these primitive terms. The primitive meaning of infinity is given in the literal meaning of one, another one – and so on, endlessly, *successively infinite*. Given that a whole is only complete if all its parts are *given at once*, the notion of a set deepens the original primitive numerical meaning of the successive infinite (traditionally known as the *potential infinite*) in the idea of *an infinite totality* (the at once infinite). The nature of ordinality and cardinality underscores this point.

Remark on mereology

Particularly in what is known as *mereology* the relationship between a whole and its parts is subjected to a set theoretical investigation.

The Polish logicist and philosopher, Stanislaw Lesniewski, under the influence of Twardowski and continuing what Husserl started in the third part of his 'Logische Untersuchungen' (*Logical Investigations*), where he analyzes the relation between a *whole and its parts*, was the first to subject this relation to a formal (set theoretical) analysis. Unfortunately, the development of mereology did not enter into an analysis of the whole-parts relation in terms of the uniqueness and mutual coherence between the aspects of number and space, for then it would have realized that set theory itself is an arithmetical theory that imitates spatial continuity. Continuous extension entails 'gaplessness' and an infinite divisibility – and when all the cohering parts are viewed collectively they constitute the original whole. Therefore the terms continuity, coherence,[7] connectedness and the whole-parts relation are interchangeable.

Within axiomatic set theory the spatial whole-parts relation is concealed in the undefined term 'set' itself! Recently Obojska introduced a single primitive mereological notion, namely *primary relation* (see Obojska 2007: pp. 644, 646). However, the term 'primary' derives from the primitive meaning of number ('first') whereas the term 'relation' derives from the primitive meaning of space (being connected). Whatever is *related is connected* and we noted that whatever is connected entails *coherence* which is synonymous with *continuity*, which in turn is synonymous with the original spatial whole-parts relation. Obojska says that Cantor's theory is *distributive* (starting with points as elements considered as a whole) whereas Lesniewski's approach is *collective* because:

> a mereological 'set' is a whole (a collective aggregate or class) composed of 'parts' and the fundamental relation is that of being a 'part' of the whole, an element of a class (Obojska 2007: p. 642).

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7   When Shapiro discusses the epistemic strategy of implicit definition in modern mathematics, he states that such a definition employs a 'direct description of the relations that hold among the places of the structure' and then mentions the second-order Peano axioms as characterising the natural-number structure while the second-order 'axioms of real analysis characterize the real-number structure' (Shapiro 2000: p. 13). He proceeds on the same page by pointing out that an intended definition must characterise at most one structure if it is categorical, i.e. if any two models of it are isomorphic to each other. However, then he adds the remark: 'Unlike isomorphism, coherence is not a rigorously defined mathematical notion, and there is no noncircular way to characterize it.'

It is therefore clear that mereology simply continues to confirm the primitive meaning of *multiplicity* and *wholeness*.

Ramifactions for axiomatic set theory

First-order predicate calculus acknowledges the necessity of employing primitive symbols in axiomatic set theory, such as connectives, quantifiers, variables and equality. An awareness of multiplicity as well as an intuition of succession are employed in this underlying discipline. Referring to quantifiers and variables clearly still exhibits the intuition of the one and the many. Consider the position within Zermelo-Fraenkel (ZF) set theory if there are more than one member present. With ZF set theory, where the general form is '*x* is a member of *y*,' both ordinality and cardinality are assumed (and successively explicated in the subsequent axioms of ZF – such as the combination of the axioms of pairing, of union and of power-set). What the axiom of power-set makes explicit is that ZF set theory depends on the original spatial whole-parts relation because it postulates for any set *a* the existence of a set whose members are just all the *subsets* of *a* (see Fraenkel et al. 1973: p. 35). The distinction between *set* and *subset* is equivalent to that of *whole* and *part*.

What is remarkable is that mathematical logic does not reflect on the fact that the terms *constants* and *variables* are embedded in the well-known philosophical problem of constancy and dynamics (persistence and change). An analysis of the meaning of number appears to need terms derived from two other unique aspects of reality, *in casu* the kinematic and physical aspects. Therefore we may now look at a further example of primitive terms by considering the uniqueness and mutual coherence between the kinematic and physical aspects, related to the problem of *constancy* and *dynamics*.

Persistence and change

Alongside the basic role of the one and the many and the whole-parts relation in our experience of and reflection on reality, the relation between *persistence* and *change* appears to be equally basic. Sometimes it is alleged that change is the only constant. Heraclitus is known for his statement that everything flows (*panta rei*) and that one cannot step into the same river twice (Diels-Kranz 1959–1960: p. 90). However, this remark is untenable because change is not identical to persistence, since it is only conceivable *on the basis* of what is constant. Without an element of persistence change cannot be established, because then the enduring subject of what is changing is absent – an insight already discovered by Plato.

De Saussure struggled with this issue in connection with the persistence of the lingual sign amidst alterations. This prompted him to articulate a correct understanding of constancy (continuity) and change:

> In the last analysis, the two facts are interdependent: the sign is exposed to alteration because it perpetuates itself. What predominates in all change is the persistence of the old substance; disregard for the past is only relative. That is why the principle of change is based on the principle of continuity (de Saussure 1966: p. 74).

Saying that 'the principle of change is based on the principle of continuity' is exactly what we have in mind with the statement that change can only be detected on the basis of something lasting, i.e. on the basis of constancy (continuity). This understanding captures a longstanding philosophical legacy, also explicitly stated by Kant when he mentions the law of the continuity of all change (he speaks of '*das Gesetz der Kontinuität aller Veränderung*'; Kant 1787-B: p. 254 [*The law of the continuity of all change*]).

The discipline of physics also had to come to terms with the relationship between continuity and change. The distinction between the kinematic and the physical (energetic) is fairly common in natural scientific circles. Max Planck, for example, sharply and correctly distinguishes between a 'mechanical' and an 'energetical' view of nature (Planck 1973: p. 65). Einstein distinguishes between the kinematic and physical aspects of reality on the basis of kinematic reversibility and physical irreversibility:

On the basis of the kinetic theory of gases Boltzman had discovered that, aside from a constant factor, entropy is equivalent to the logarithm of the 'probability' of the state under consideration. Through this insight he recognized the nature of courses of events which, in the sense of thermodynamics, are 'irreversible'. Seen from the molecular-mechanical point of view, however, all courses of events are reversible (Einstein 1959: p. 43).[8]

Alongside Planck and the subsequent orientation of twentieth-century physics, Einstein here rejects the *mechanistic* orientation of classical physics – which attempted to reduce all (irreversible) physical processes to (reversible) mechanical movements (see also Planck 1973: p. 53). Janich distinguishes between 'kinematic [*phoronomischer* (*im folgenden kinematisch genannt*)] and dynamic statements' (Janich 1975: pp. 68–69) and in a more recent work he positions this distinction within the broader context of the transition from the nineteenth to the twentieth century occurring within mathematics, physics, chemistry and logic (see Janich 2009: p. 140 ff.).[9]

The reaction of twentieth-century physics to the dominant mechanistic tendency of classical physics therefore occurred on the basis of acknowledging the uniqueness of and the coherence between *constancy* and *change*, aptly captured in the first main law of thermodynamics, the law of energy conservation. This law is best described as the law of *energy constancy* (a kinematic analogy on the law-side of the physical aspect). Whenever energy operates it causes *changes* solely detectable on the basis of constancy. The concept of *mass* in contemporary physics designates a *quantity of matter*, thus revealing the meaning of the physical through the *coherence* between the physical aspect (matter) and number (the quantitative mode). Similarly, the *field* concept in physics reveals the coherence between the physical aspect and the spatial aspect. These three foundational analogies highlight the general idea of the structure of a modal aspect, because it shows that the meaning of the physical aspect only comes to expression in its coherence with the other (foundational) aspects.

The terms *persistence*, *uniform* and *constancy* are primitive and irreducible – they recur in the kinematic notion of rectilinear motion and constitute the crux of Galileo's law of inertia. For this reason one cannot speak about a cause of motion, it is only meaningful to refer to the cause of a *change* in motion (von Weizsäcker 2002: 172). Kant provided a formulation in which the meaning of the law of energy constancy is anticipated: 'Amidst all changes in material bodies the quantity of matter on the whole remains the same' (Kant 1786: 116 – see von Weizsäcker 2002: 197).

Persistence over time derives from intuiting the kinematic meaning of rectilinear movement (uniformity/constancy) and it underlies the idea of *identity*. Quine is justified in his claim that 'identity' is 'such a simple and fundamental idea that it is hard to explain otherwise than through mere synonyms' (quoted by Grau 1999: 77). Quine clearly discerned the core meaning of the kinematic mode (explicable only by synonyms), and the fact that our awareness of the identity of entities crucially depends upon an extended use of the core meaning of the kinematic aspect.[10]

We may relate our awareness of identity to the distinctness of different entities and to the human ability to identify what is distinct and to distinguish it from other identifiable entities. This relates to the core meaning of the logical–analytical aspect that rests on the two legs of identification and distinguishing.

Analysis: similarities and differences

Logical analysis proceeds by means of *identification* and *distinguishing* and it is always accomplished on the basis of *similarities* and *differences*, as captured in Figure 2:

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8   In a different context Einstein advances a negative appreciation of 'the mechanistic framework of classical physics' (see Einstein 1985: p. 146). Yet his discussion of simultaneity within his special theory of relativity takes place in the Kinematic Section of his original 1905 article (see Einstein 1985: p. 25 ff.).

9   Yourgrau frequently refers to this distinction in his work on Gödel and Einstein: (Yourgrau 2005: pp. 36, 114, 115, 123, 129, 133, 142).

10 Both Grau and Quine view identity in relation to entity: 'No identity without entity. Nonentities are not there to be the same or different' (see Grau 1999: p. 80).
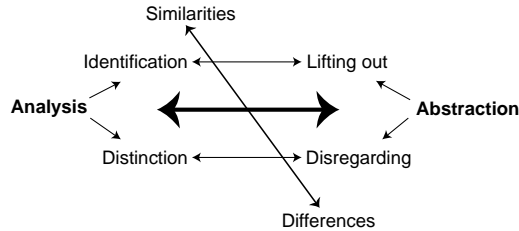
Figure 2.

Given that similarities presuppose differences and because differences presuppose similarities, the meaning of analysis displays an element of *circularity*. We should keep in mind that the ability to identify and distinguish embodies the acquisition of concepts, for a concept unites a multiplicity of (logically identified and distinguished) features in its conceptual unity. When primitives are at stake the challenge is not to get outside the circle, but to get *inside* it!

Acts of logical-analytical identification and distinguishing are intimately related to the lingual terms used to designate what has been identified and distinguished – which prompts us briefly also to pay attention to the meaning of meaning.

The meaning of meaning

Semantics, as a subdiscipline of general linguistics, had to accept 'meaning' as such as a primitive term.[11] The distinction made by Immanuel Kant between analytic and synthetic propositions (cf. Kant 1787: p. 10 ff.) may tempt one to define *synonymity* in terms of *analyticity*. If two sentences have the same meaning in the sense of entailing each other, they imply each other *analytically*. Quine identified the circularity of such an attempt, since analyticity is defined in terms of meaning,[12] whereas meaning (in this case: similarity of meaning = synonymity) is defined in terms of analyticity. Fodor highlights an element of this insight in pointing out that the meaning of the sign mode cannot be understood apart from an understanding of the meaning of the logical–analytical mode:

> The goal we have been pursuing is the traditional one of reducing meaning to some more basic and better understood entity. But analyticity is too intimately related to meaning to provide such a reduction. In fact, as far as anyone knows, there is no meaning-independent way of characterizing either analyticity or meaning (Fodor 1977: p. 43).

In respect of primitive terms there are only two options: either provide a synonym or reduce what is primitive to something else.

The coherence between primitive terms

In Figure 1 the analogical connections between different aspects are captured. When two aspects are similar in that respect in which they differ, or different in that respect in which they are similar, we meet a *modal analogy* – see Figure 3 regarding the spatial analogy of distance within the social aspect. [The President of a country and its bodyguard are in *spatial proximity* but in terms of their respective social positions *far apart*.]

It is clear that every possible modal analogy always presupposes the original meaning of another aspect distinct from the one in which it appears as an analogy. However, the mere fact that, through their analogies, the inter-modal connections between aspects come to expression, as such, does not decide which one of the following four options is correct:

(1) aspects refer to aspects;

(2) the meaning-nucleus of an aspect refers to other aspects;

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11 Analysing the lingual subject–object relation as well as the multiple meaning-nuances belonging to the semantic domain (a spatial analogy within the structure of the sign-mode) of the word meaning will not be further discussed in this context.

12 A sentence is supposed to be analytically true if it is true only on the basis of its meaning.

Figure 3.

(3) aspects refer to the meaning-nuclei of other aspects; or
(4) Meaning-nuclei refer to meaning-nuclei.

   Although the heading of this section, which refers to the 'coherence between primitive terms', may suggest option (4), it is not correct because option (3) appears to capture the intended coherence. The term employed to designate the full structure of an aspect normally comes first because it qualifies the composite phrase through which a modal analogy is designated. In our example of *social distance*, the term *social* represents the entire social aspect and the term *distance* (one-dimensional extension) highlights the core meaning of the spatial aspect. Similar composite phrases are *economic growth* and *biotic growth*, *aesthetic expression* and *lingual expression*, *cultural development* and *biotic development*, and so on.

   The implication is that one should not speak of the 'coherence of irreducibles' but rather of the inter-modal coherence between different aspects (where an aspect is understood as summarised in Figure 1). Given that the core meaning of an aspect is conceptually indefinable, different nuclei cannot be defined in terms of each other.

   We noted that one needs an appropriate term to designate what cannot be defined and this word must reflect what is intuitively evident about the uniqueness of the aspect under consideration. The important perspective is that the meaning of an aspect only comes to expression in its *coherence* with other aspects – an insight that we shall briefly elucidate with reference to logic (number and space), physics and the concept of law.

Logic, number and space
Hilbert correctly emphasises that logic and arithmetic cannot analyse their respective fields of investigation without acknowledging the interconnections between these domains. In 1913, Hilbert wrote:

> Only when we analyze attentively do we realize that in presenting the laws of logic we already have had to employ certain arithmetical basic concepts, for example the concept of a set and partially also the concept of number, particularly as cardinal number [*Anzahl*]. Here we end up in a vicious circle and in order to avoid paradoxes it is necessary to come to a partially simultaneous development of the laws of logic and arithmetic (Hilbert 1913a: pp. 245–246; see also Quine 1970: p. 88).

   Of course, it is only possible to reflect on such coherences once what is unique has been established on the basis of an intuitive insight. In our earlier brief discussion of intuitive insight we mentioned that Yourgrau explained that Gödel 'insisted that to know the primitive concepts, one must not only understand their relationships to the other primitives but must grasp them on their own, by a kind of "intuition"' (Yourgrau 2005: p. 169).

   The general philosophical issue could be designated as that concerning 'the coherence of irreducible aspects' – one of the classical challenges to philosophical thinking.

   The examples considered thus far all underscore the importance of such an intuitive, immediate insight into what is basic and primitive. The accompanying perspective explores the 'relationships to the other primitives,' as Yourgrau stated it. We may put it in stronger terms: whatever is unique and primitive only reveals its meaning in and through its coherence with all the other primitives.

   This point is also illustrated in David Hilbert's remark about 'analogies' between number and space and his simultaneous distinction of these two aspects ('a diversity').

In his 'On the Number Concept' [*Über den Zahlbegriff*], published in volume 8 of *Jahresbericht der Deutschen Mathematiker-Vereinigung*, David Hilbert proceeds from a clear insight into the *uniqueness* (diversity) of and the *coherence* between the objects of arithmetic and the axioms of geometry: 'When we observe within the literature numberless works on the principles of arithmetic and on the axioms of geometry and mutually compare them, then we see, alongside many analogies and connections between these fields of study, indeed, in respect of their method of investigation, a diversity' (Hilbert 1913b: p. 237).[13]

Defining what is primitive: only two options

We noted that when an attempt is made to define something truly primitive, only two options exist: either a (partial) synonym is provided or a distinct primitive term, belonging to a different aspect, is used, to which it is then reduced. Ultimately, primitive aspectual terms reside in a multiplicity of sphere-unique aspects.

In many of the examples discussed thus far the irreducibility of the aspects of number, space, the kinematic and the physical featured. All of the other aspects contain analogies of these aspects within their own modal structure, thus supporting the conjecture that everything (aspect) is unique and that everything (all aspects) mutually cohere.

The last remark entails that all the academic disciplines have to account for the specific meaning of analogical terms appearing within the structure of the aspect that delimits their particular fields of study. For example, whereas mathematical space and physical space are extended (the similarity between them), the former is both continuous and infinitely divisible, while the latter is neither infinitely divisible (owing to the quantum-structure of energy) nor continuous (the difference).[14] The history of the concept of matter reflects an overemphasis of the aspects that are foundational to the physical aspect: the Pythagoreans reduced matter to number (everything is number). Then, with the discovery of irrational numbers, a switch was made to space (which lasted up to Descartes and Kant). Since Galileo the mechanistic main tendency of modern physics, as noted earlier, reduced matter to motion (up to Heinrich Hertz). It was only since the discovery of radioactivity and Planck's *Wirkungsquatum* that twentieth-century mathematics acknowledged the *physical nature* of matter. However, explaining how the history of the other natural sciences and the humanities reveals the one-sided (and reductionist) effect of overemphasising particular analogical moments exceeds the aim of our current discussion of the problem of primitive terms.

Concluding remark

Not only does an analysis of the phenomenon of primitive terms illustrate the limits of conceptual understanding, it also highlights the inevitable acknowledgment of the mutual coherence of aspects with their respective primitive terms. Furthermore, it also challenges every academic discipline to generate an account of the way in which the meaning of what is unique and primitive comes to expression in its coherence with other aspects. The overestimation of rational understanding found in all rationalistic trends of thought ultimately underscores an embarrassing situation: only by using primitive terms exceeding the grip of rational understanding is it possible to reach rational understanding at all. Just recall Weyl's words: 'It must have been hard on Hilbert, the axiomatist, to acknowledge that the insight of consistency is rather to be attained by intuitive reasoning which is based on evidence and not on axioms' (Weyl 1970: p. 269).

At the same time the problem of the coherence of distinct aspects (embracing what is primitive to them) challenges all the special sciences to give an account of their unavoidable employment of analogical basic concepts – a theme exceeding the aim of the current investigation.

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13 '*Wenn wir in der Literatur die zahlreichen Arbeiten über die Prinzipien der Arithmetik und über die Axiome der Geometrie überschauen und mit einander vergleichen, so nehmen wir neben zahlreichen Analogien und Verwandtschaften dieser beiden Gegenstände doch hinsichtlich der Methode der Untersuchung eine Verschiedenheit wahr*' (Hilbert 1913b: p. 237).

14 Already in 1925 Hilbert has mentioned this difference (see Hilbert 1926: p. 164).

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